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LETTER TO THE EDITOR

The superfluid phase transition in neutron star matter

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Abstract. The Ginzburg–Landau free energy describing p wave paired neutron star matter is used to derive renormalisation group equations. The absence of infrared stable fixed points indicates a first order phase transition.

It has been shown by Jones *et al* (1976) and Bailin *et al* (1977) that first order phase transitions occur from the normal phase into superfluid phases of ^{3}He when account is taken of fluctuations by means of the renormalisation group. Neutron star matter in the density range $1\text{–}8\times10^{14}\ \mathrm{g\ cm^{-3}}$ is believed to be a p wave superfluid like ^{3}He but with a strong spin–orbit force producing pairing in a $J=2$ state. (Hoffberg *et al* 1970 and references therein.) Modern calculations suggest that the $^3P_2$ pairing is due to tensor rather than spin–orbit forces (Takatsuka 1972). In this note we investigate the possibility that the superfluid phase transition in neutron star matter is also first order because of fluctuations.

The order parameter for neutron star superfluidity is a complex 3×3 matrix A_{ij}, which because of the $J=2$ nature of the pairing is traceless and symmetric. The Ginzburg–Landau bulk free energy density is of the form

$$F_{\mathrm{B}}=r\ \mathrm{tr}(AA^*)+\tfrac{1}{4}u|\mathrm{tr}\ A^2|^2+\tfrac{1}{4}v(\mathrm{tr}\ AA^*)^2+\tfrac{1}{4}w\ \mathrm{tr}\ A^2A^{*2}, \tag{1}$$

(Sauls and Serene 1978), where r vanishes at the transition temperature in the mean field approximation. The most general bending (or strain) free energy density consistent with symmetry under simultaneous spin and space rotations, and with phase symmetry, is

$$F_{\mathrm{S}}=\alpha^{-1}\partial_iA^*_{ki}\partial_jA_{kj}+\epsilon_{ijk}\partial_jA^*_{pk}\epsilon_{ilm}\partial_lA_{pm} \tag{2}$$

apart from total divergences.

We shall construct renormalisation group equations in the form given by 't Hooft (1973) and the calculations are greatly simplified by the fact that we may use massless propagators ($r=0$). The appropriate propagator is

$$\begin{aligned}P_{kl,ij}(q)&=(1/2q^2)[\delta_{ki}\delta_{lj}+\delta_{kj}\delta_{li}+2(\lambda-d)d^{-2}\delta_{kl}\delta_{ij}]\\&\quad+[\lambda/4(q^2)^2][\delta_{ik}q_lq_j+\delta_{lj}q_kq_i+\delta_{kj}q_lq_i+\delta_{li}q_kq_j-4d^{-1}(\delta_{kl}q_iq_j+\delta_{ij}q_kq_l)],\end{aligned} \tag{3}$$

where the necessary symmetrisation and subtraction of traces has been done to ensure that no spurious contributions arise from other than $J=2$, we work in $d=4-\epsilon$ dimensions of space, and

$$\lambda \equiv \alpha - 1. \tag{4}$$

(The bending energy of equation (2) extended to d dimensions requires A_{ij} to be a $d\times d$ matrix.)

An order ϵ^2 calculation shows that the parameter λ has the Callan–Symanzik function (linearised about $\lambda=0$)

$$(16\pi^2)^2\beta_\lambda = \tfrac{1}{2}\lambda(6u^2+3v^2+\tfrac{11}{12}w^2+\tfrac{5}{3}uw+vw). \tag{5}$$

Since the quadratic function of u, v and w is positive definite, λ has an infrared stable fixed point at $\lambda=0$. Accordingly, we shall take $\alpha=1$ in subsequent calculations.

When $\alpha=1$, the propagator simplifies to a scalar meson propagator, and in general we may take A_{ij} to be an $n\times n$ matrix with n different from d. The corresponding symmetrised traceless propagator is

$$P_{kl,ij}(q) = [1/2(q^2+r)][\delta_{ki}\delta_{lj}+\delta_{kj}\delta_{li}-(2/n)\delta_{kl}\delta_{ij}]. \tag{6}$$

At order ϵ, the Callan–Symanzik functions for the coupling constants u, v and w are

$$\begin{aligned}
&32\pi^2(\beta_u+\epsilon u)\\
&\quad=(n+2)(n-1)u^2+(n^3+8n^2-16n+8)(w^2/4n^2)\\
&\qquad+12\,uv+(n^2+3n-2)(2uw/n),\\
&32\pi^2(\beta_v+\epsilon v)\\
&\quad=8u^2+(n^2+n+6)v^2+(n^3+6n^2-16n+16)(w^2/2n^2)\\
&\qquad+8uv+8uw+(n^2+2n-4)(2vw/n),\\
&32\pi^2(\beta_w+\epsilon w)=-(n+4)(n-2)(w^2/2n)-8uw+12vw.
\end{aligned} \tag{7}$$

These equations have the infrared unstable gaussian fixed point

$$u^*=v^*=w^*=0 \tag{8}$$

and the symmetric fixed point

$$u^*=w^*=0, \qquad v^*=32\pi^2\epsilon/(n^2+n-2+8) \tag{9}$$

(The complex traceless symmetric matrix A_{ij} has n^2+n-2 real entries.) However, general arguments of Brézin *et al* (1974) show that this latter fixed point is unstable for more than $4+O(\epsilon)$ real fields, unless the original free energy is completely symmetric ($u=w=0$). Since we are interested in $n>2$, this fixed point is unstable. Moreover, there are no other fixed points when $n=3$, and so for the case of physical interest the renormalisation group equations have no infrared stable fixed points, and we expect a first-order phase transition. (See for example Halperin *et al* 1974.)

Whether and when a neutron star goes through the superfluid phase transition depends on the value of T_c (which is notoriously difficult to estimate reliably for superfluid transitions.) The estimate of Hoffberg *et al* (1970) gives $T_c\sim 10^9$ K, one order of magnitude larger than estimated maximum temperatures for neutron star cores more than 10^3 years old, though very young neutron stars may attain temperatures up to three orders of magnitude greater than T_c. When the first-order superfluid transition occurs we expect a jump in the order parameter squared determined by the parameter

ϵ_c which controls the width in temperature of the critical region in units of T_c (Ginzburg 1960). Using the propagator of equation (6) with $n = 3$ we estimate

$$\epsilon_c^{1/2} = (1/4\pi)(u + 3v + \tfrac{11}{12}w). \tag{10}$$

With weak coupling BCS values of u, v and w and $T_c/T_F \approx 4 \times 10^{-3}$, equation (10) gives

$$\epsilon_c \sim 10^{-4}.$$

Since ϵ_c is proportional to $(T_c/T_F)^4$, a somewhat larger value of T_c/T_F would imply a much larger value of ϵ_c, and therefore of the order parameter. Maxwell *et al* (1977) have observed that an important contribution to the rate of cooling of a neutron star is the neutrino emission induced by the pion condensate, and that this emission is inhibited in the superfluid ordered phase. The existence of a first-order transition to this phase means that the cooling is delayed by the latent heat of the transition and also by the discontinuous development of a non-zero order parameter. Thus the importance of this mechanism is highly sensitive to the value of T_c/T_F, for the reason given above.

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